S. S. Jain Subodh Management Institute

MBA IInd Semester, (Model Paper & Suggested Answers)

Subject: Operation Research

Paper Code: M-208

Time: 1 Hour **Max Marks: 10**

Q1. What is operational Research? Describe various techniques used in operational Research. What is O.R.?

Q2. A company has four factories F_1, F_2, F_3 and F_4 manufacturing the same, product. Production and raw material costs differ from factory to factory and are given in the following table in the first two rows. The transpiration costs from the table give the sales depot, S_1, S_2, S_3 are also given. The last two columns in the table give the sales price and the total requirements at each depot. The production capacity of each factory is given in the last row.

					Sales Price per unit	Requirement
	F_1	F_2	F_3	F_4		
Production Cost/Unit	15	18	14	13		
Raw material Cost/Unit	10	9	12	9		
S1	3	9	5	5	34	80
S2	1	7	4	5	32	120
Transportation Cost/Unit S3	5	8	3	6	31	150
	10	150	50	100		

Determine the most profitable production and distribution schedule and the corresponding profit. The surplus production should be taken to yield zero profit.

Q3. A Company is producing a single product and is selling it through five agencies situated in different cities. All of a sudden, there is a demand for the product in another five cities not having any agency of the company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to needy cities in such a way that the travelling distance is minimized. The distances (in kms) between the surplus and deficit cities are given in the following distance-matrix.

Surplus Cities / Deficit Cities	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Determine the optimum assignment schedule.

Suggested Answers

Answer 1:

Operational Research can be considered as being the application of scientific method by inter-disciplinary teams to solve problems involving the control of organized (man-machine systems) so as to provide solutions which best serve the purposes of the organization as a whole.

Features of OPERATIONAL RESEARCH (OR)

- O.R is developed “to ensure reduction in costs.
- O.R is a scientific approach to problem solving.
- For executive decision making which requires formulation of mathematical, economic and statistical models for decision making and controlling.

Advantages & Limitations of Operations Research

Advantages

1. Batter Control
2. Better Systems
3. Better Decisions
4. Better Co-ordination

Limitations

1. Dependence on an Electronic Computer
2. Non-Quantifiable Factors
3. Money and Time Costs
4. Implementation

Such as better customer satisfaction, skill attitude of worker, initiative of management etc.

Various O.R. methods & technique have been developed and applied to solve different types of problems. These models can be grouped into several basic forms. This classification of O.R. problem forms the basis for the structure. A brief account of the important O.R. models is given blow :

Techniques of Operations Research

1. Linear Programming
2. Transportation Problem

3. Assignment Problem
4. Queuing Theory
5. Game Theory
6. Inventory Control Models
7. Decision Theory
8. Network Analysis
9. Simulation
10. Replacement Problems
11. Sequencing

<u>A brief account of some of the important O.R. techniques is given below:-</u>

1. Linear Programming: Linear Programming is method of finding a maximum or minimum value of a function that satisfies a given set of conditions called constraints. A constraint is one of the inequalities in a linear programming problem. The solution to the set of constraints can be graphed as a feasible region.
2. Transport problem: A transport problem (TP) consists of determining how to route products in a situation where there are several supply locations and also several destinations in order that the total cost of transportation.
3. Assignment: assignment is a technique to find out lowest possible allocable cost of the project by assigning the best possible least costs to the jobs and workers.
4. Waiting or Queuing theory: interruptions can be occurred due to many reasons. These cause waiting line problem. Waiting line theory aims in minimizing the cost of both servicing and waiting.
5. Game theory: these are model which arise when two or more people are competing. Game model is used to determine the optimum strategy in a competitive situation.
6. Inventory model: regarding the holding or sorting the resources. How much to acquire and when to acquire?
7. Decision theory: concerned with making decisions under the conditions of certainty, risk and uncertainty.
8. Network analysis: determination of an optimum sequence of performing certain operations concerning some jobs to minimize overall time and money.
9. Simulation: technique of testing a model which resembles a real life situation. Imitate an operation before actually performing it.

10. Replacement theory: concerned with situation that arise when some items need replacement because the same may be deteriorated. The model concerned with prediction of replacement cost and determination of the most economic replacement policy.

11. Sequencing: it is the selection of an appropriate order in which a number of jobs (operations) can be assigned to a finite number of service facilities (Machines or equipments) so as to optimize the outputs in terms of time, cost or profit.

Answer 2

Solution. The profit matrix is derived by the equation :

Profit = Sales price—production cost—raw material cost —transportation cost

TABLE I (*Profit Matrix*)

SALE DEPOTS / FACTORIES	S_1	S_2	S_3	AVAILABILITY
F_1	6	6	1	10
F_2	–2	–2	–4	150
F_3	3	2	2	50
F_4	8	5	3	100
REQUIREMENT	80	120	150	310 350

Total requirements = 80 + 120 + 150 = 350

Total availabilities = 10 + 150 + 50 + 100 = 310

Dummy factory to 'produce' 350 – 310 = 40 units

TABLE II

SALE DEPOTS / FACTORIES	S_1	S_2	S_3	AVAILABILITY
F_1	6	6	1	10
F_2	–2	–2	–4	150
F_3	3	2	2	150
F_4	8	5	3	100
(DUMMY) F_5	0	0	0	40
REQUIREMENT	80	120	150	350

TABLE III (*Relative Loss Matrix*)

SALE DEPOTS / FACTORIES	S_1	S_2	S_3	AVAILABILITY
F_1	$8 - 6 = 2$	2	7	10
F_2	$8 - (-2) = 10$	10	12	150
F_3	5	6	6	50
F_4	0	3	5	100
(DUMMY) F_5	8	8	8	40
REQUIREMENT	80	120	150	350

Vogel's method is now applied to derive the initial feasible solution as shown in following *Table IV*.

TABLE IV

SALE DEPOTS / FACTORIES	S_1	S_2	S_3	AVAILA-BILITY
F_1	2	2 (10)	7	10
F_2	10	10 (40)	12 (110)	150
F_3	5	6 (50)	6	50
F_4	0 (80)	3 (20)	5	100
(DUMMY) F_5	8	8	8 40	40
REQUIREMENT	80	120	150	350

Since there are 7 (*i.e.*, $m + n - 1$) allocations, the initial solution is straightway tested for optimality in *Table V* using MODI Method. Determine row and column numbers u_i ($i = 1, 2, 3, 4$) and v_j ($j = 1, 2, 3$) by using the formula $c_{ij} = u_i + v_j$ for occupied cells, as shown in *Table V*. The opportunity costs for all the unoccupied cells are calculated by using formula $\Delta_{ij} = c_{ij} - (u_i + v_j)$ and are also shown in the following *Table V*.
Since the cell (F_3, S_3) has the negative opportunity cost -2, it is admitted as an entering variable (cell) in the solution. On drawing closeid loop or path, we find that 50 units should be shipped from (F_2, S_2) or F_2, S_3 to (F_3, S_2). This yields the new solution as given in *Table VI*.

TABLE V

Sale Depot / Factories	S_1	S_2	S_3	Availa-bility	Row number
F_1	2 / 3	2 (10)	7 / 3	10	$u_1 = 2$
F_2	10 / 3	10 (40) +	12 (110) −	150	$u_2 = 10$
F_3	5 / 2	6 (50) −	6 / −2 +	50	$u_3 = 6$
F_4	0 (80)	3 (20)	5 / 0	100	$u_4 = 3$
Dummy	8 / 5	8 / 2	8 (40)	40	$u_5 = 0$
Requirement	80	120	150	350	
Column Number	$v_1 = -3$	$v_2 = 0$	$v_3 = 2$		

Since the cell (F_3,S_3) has the negative opportunity cost -2, it is admitted as an entering variable (cell) in the solution. On drawing closed loop or path, we find that 50 units should be shipped from (F_2,S_3) or (F_3,S_2) to (F_3,S_3) This yield the new solution as given in the following *Table VI*.

TABLE VI

Sale Depot / Factories	S_1	S_2	S_3	Availa-bility	Row number
F_1	2 / 3	2 (10)	7 / 3	10	$u_1 = 4$
F_2	10 / 3	10 (90)	12 (60)	150	$u_2 = 12$
F_3	5 / 4	6 / 2	6 (50)	50	$u_3 = 6$
F_4	0 (80)	3 (20)	5 / 0	100	$u_4 = 5$
Dummy	8 / 5	8 / 2	8 (40)	40	$u_5 = 8$
Requirement	80	120	150	350	
Column Number	$v_1 = -5$	$v_2 = 2$	$v_3 = 0$		

The solution given above in *Table VI* is optimum, since there is no negative opportunity cost in the unoccupied cells. The profit associated with this solution is :

$$80(8) + 10(6) + 90(-2) + 20(5) + 60(-4) + 50(2) = \text{Rs. } 480.$$

Answer 3

Solution. *Step 1.* Subtracting the smallest element of each row from every element of the corresponding row, we get the adjoining reduced matrix (*Table 1*).

TABLE 1

Deficit Cities / *Surplus Cities*	*I*	*II*	*III*	*IV*	*V*
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Step 2. Subtracting the smallest element of each column from every element of the corresponding column to get the following reduced matrix :

TABLE 2

Deficit Cities / *Surplus Cities*	*I*	*II*	*III*	*IV*	*V*
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

Step 3.

TABLE 3

Deficit Cities / *Surplus Cities*	*I*	*II*	*III*	*IV*	*V*
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

Step 4 (*a*) Select the smallest element (15), among all *uncovered* elements of the *Table* 3, and

(*b*) Subtract this value (15) from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines.

(c) leaving the remaining elements unchanged. The following matrix (*Table*4) is obtained.

Step 5 Again, proceeding exactly in the like manner of *step* , we reach the final matrix (*Table*5).

TABLE 4

Surplus Cities \ Deficit Cities	*I*	*II*	*III*	*IV*	*V*
A	15	0	20	15	0
B	15	15	0	10	0
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

TABLE 5

Surplus Cities \ Deficit Cities	*I*	*II*	*III*	*IV*	*V*
A	15	~~0~~	20	15	[0]
B	15	15	[0]	10	~~0~~
C	15	[0]	20	15	5
D	[0]	15	20	~~0~~	5
E	5	~~0~~	10	[0]	~~0~~

It may be noted that in *table* 5 there are no remaining zeros, and every row and column has an assignment. Since no two assignments are in the same column (they cannot be, if the procedure has been correctly followed, the 'zero-assignment' is the required optimum solution.

From original matrix, the optimum assignment schedule is therefore :

Route	*Distance (kilometres)*
A—V	200
B—III	130
C—II	110
D—I	50
E—IV	80
Minimum distance travelled	570 kilometres

Remark. It is possible to quickly reach the final *Table* 5, if we first apply *step 2* and then *step 1* on the original matrix. The reader is advised to solve this example again and to verify this fact.